

University of Bahrain
College of Information Technology
Department of Computer Science
ITCS251/252 Discrete Structure I
2nd Semester 2011/2012
Quiz #1
(Section 4)

9.5

ID:

Name:

Q1. (10marks) Let $A = \{\emptyset, \{1\}, 1\}$ and $B = \{x \in \mathbb{R} \mid x^2 - 3x = 0\}$. Find

- a) $A \cup B$
- b) $A \cap B$
- c) $P(A)$
- d) $A - B$
- e) A partition of A

$$A = \{\emptyset, \{1\}, 1\}$$

$$B = \{x \mid x(x-3) = 0\}$$

$$x = 0 \quad \text{or} \quad x = 3$$

$$B = \{0, 3\}$$

$$a) A \cup B = \{\emptyset, \{1\}, 1, 0, 3\} \quad \checkmark$$

$$b) A \cap B = \emptyset \quad \checkmark$$

$$c) P(A) = \{\emptyset, \{\emptyset\}, \{\{1\}\}, \{1\}, \{\emptyset, \{1\}\}, \{\emptyset, 1\}, \{\{1\}, 1\}, \{\emptyset, \{1\}, 1\}\} \quad \checkmark$$

$$d) A - B = \{\emptyset, \{1\}, 1\} = A \quad \checkmark$$

$$e) \text{ Let } B = \{\emptyset, \{1\}\}$$

$$\text{Let } C = \{1\}$$

$$B \cup C = \{\emptyset, \{1\}, 1\} = A \quad \checkmark$$

$$B \cap C = \emptyset$$

$\therefore B, C$ partition of A

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Quiz #2

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1. (4 marks) Show whether or not the following is a tautology

$$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow (r \wedge q)) \equiv \text{true}$$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$r \wedge q$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \rightarrow (r \wedge q)$	(X)
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	F	T	F	F	F	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	F	F	F	T	T
F	F	T	T	T	F	T	T	T
F	F	F	T	T	F	T	T	T

2. (1 marks) Find truth-value of "If 9 is even then 8 is odd" \equiv true

\therefore (X) is a tautology

3. (3 marks) Consider the statement

n is prime only if n is odd or n is two

- Write the statement in symbolic form
- Write the negation of the statement
- Write the contrapositive statement

let p: n is prime
q: n is odd
r: n is two

i) $(q \vee r) \rightarrow p \equiv \neg(q \vee r) \vee p \equiv (\neg q \wedge \neg r) \vee p$

ii) $\sim [(\neg q \wedge \neg r) \vee p] \equiv \sim(\neg q \wedge \neg r) \wedge \sim p \equiv (q \vee r) \wedge \sim p$

- n is odd or it is two, and n is not prime.

iii) $\sim p \rightarrow \sim(q \vee r) \equiv \sim p \rightarrow (\neg q \wedge \neg r)$

- if n is not odd and it is not two, then n is not prime.
not

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Quiz #3

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Q1. (4 marks) Determine whether or not the following is valid.

$$\begin{array}{l} \textcircled{1} p \rightarrow q \\ \textcircled{2} \sim q \vee r \equiv q \rightarrow r \\ \therefore p \rightarrow r \end{array}$$

from (1) and (2)

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

\therefore Valid

Q2. (6 marks) Determine whether the following argument is valid or not.

1. If you study, then you pass.
 2. If you do not study, then I will feel bad.
 3. I will not feel bad
- \therefore you pass

Let: s: you study
p: you pass
b: feel bad

$$\begin{array}{l} \textcircled{1} s \rightarrow p \\ \textcircled{2} \sim s \rightarrow b \\ \textcircled{3} \sim b \\ \hline \therefore p \end{array}$$

from (2) and (3)

$$\begin{array}{l} \sim s \rightarrow b \\ \sim b \\ \hline \end{array}$$

$\textcircled{4} \therefore s$

from (1) and (4)

$$\begin{array}{l} s \rightarrow p \\ s \\ \hline \therefore p \end{array}$$

\therefore Valid

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Quiz #4

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Q1. (6 marks)

Let $D(x)$: x is a doctor
 $T(x)$: x is trained

Convert the following in symbolic using above predicates and quantifiers.

- a) All doctors are trained
- b) Some doctors are trained
- c) No doctor is trained
- d) Not all doctors are trained

$$\forall x D(x) \rightarrow T(x)$$

a) $\forall x D(x) \rightarrow T(x)$

b) $\exists x D(x) \wedge T(x)$

d) $\sim (\forall x D(x) \rightarrow T(x)) \equiv \exists x \sim (\sim D(x) \vee T(x)) \equiv \exists x D(x) \wedge \sim T(x)$

c) $\forall x \sim (D(x) \rightarrow T(x)) \equiv \sim (\exists x (D(x) \rightarrow T(x)))$
 $\forall x D(x) \rightarrow \sim T(x)$

Q2. (4 marks) Find the truth value for each of the following.

- a) $\exists x \in \mathbb{Z}, x^2 = 4$
- b) $\forall x \in \mathbb{Z}, x^2 = 2$

a) for $x = 2 \in \mathbb{Z} \Rightarrow 2^2 = 4 \Rightarrow 4 = 4$ is true
 $\therefore \exists x \in \mathbb{Z}, x^2 = 4$ is true.

b) for $x = 3 \in \mathbb{Z} \Rightarrow 3^2 \neq 4 \Rightarrow 9 \neq 4$ is false
 $\therefore \forall x \in \mathbb{Z}, x^2 = 2$ is false.

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Quiz #5
(Section 1)

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Q1. (5marks). Prove that for all integers a, b, c , if a divides b then a divides bc .

$$\forall a, b, c \in \mathbb{Z} \quad b = ak_1 \longrightarrow bc = ak_2$$

to assume: $b = ak_1, k_1 \in \mathbb{Z}$

to conclude: $bc = ak$

let: $b = ak_1, a, b, k_1 \in \mathbb{Z}$

by multiplying by $c: c \in \mathbb{Z}$

$$bc = ak_1 c$$

$$bc = ak, k = k_1 c \in \mathbb{Z}$$

\Rightarrow Hence proved.

Q2. (5 marks) Determine whether the following statement is true. If true, then prove it else give counter example.

a) For all integers n , if $n^2 + 1$ is odd then n is odd.

b) For all integers n , $\frac{n}{n-1}$ is not integer.

a) the statement is false:

counter example:

$$\text{for } n^2 + 1 = 17 \text{ (odd)}$$

$$n^2 = 16$$

$$n = \pm 4 \text{ (even)}$$

2.5

b) the statement is false

counter example:

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for $n = 0$

$$\frac{n}{n-1} \Rightarrow \frac{0}{0-1} = \frac{0}{-1} = 0 \in \mathbb{Z}$$

Q: A sequence a_1, a_2, a_3, \dots is defined as

$$a_1 = 3$$

$$a_k = 7a_{k-1}, \quad k \geq 2$$

Show that:

$$a_n = 3 \cdot 7^{n-1}, \quad n \geq 1$$

given: $a_1 = 3$

$$a_k = 7a_{k-1}, \quad k \geq 2$$

To prove: $a_n = 3 \cdot 7^{n-1}, \quad n \geq 1$

let $p(n): a_n = 3 \cdot 7^{n-1}, \quad n \geq 1$

Basis: To show $p(1)$:

$$\text{for } n=1 \Rightarrow a_1 = 3 \cdot 7^{1-1} = 3 \cdot 1 = 3 \quad (\text{True from given})$$

$\therefore p(1)$

Assumption: let:

$$p(k): a_k = 3 \cdot 7^{k-1}, \quad k \geq 1$$

Induction: To show:

$$p(k+1): a_{k+1} = 3 \cdot 7^k, \quad k \geq 1$$

from given $a_k = 7a_{k-1}$

$$\text{for } k=k+1 \Rightarrow a_{k+1} = 7a_k$$

$$= 7 \cdot 3 \cdot 7^{k-1} \quad (\text{from assumption}), \quad k \geq 1$$

$$= 3 \cdot 7^k, \quad k \geq 1$$

$\therefore p(k+1)$

Hence proved

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Q. Let $f: \mathbb{Z} \rightarrow \mathbb{Z}$ defined by: $f(x) = x^2$

Is f

a) one-to-one, b) on-to

a) f is one-to-one $\longleftrightarrow \forall x_1, x_2 \in \mathbb{Z} \quad f(x_1) = f(x_2) \longrightarrow x_1 = x_2$

to assumes $f(x_1) = f(x_2)$

to concludes $x_1 = x_2$

Let $x_1, x_2 \in \mathbb{Z}$ and $f(x_1) = f(x_2)$

$$x_1^2 = x_2^2$$

$$x_1 = \pm x_2$$

Let $x_1 = 2 \in \mathbb{Z}$, $x_2 = -2 \in \mathbb{Z}$

$$\text{L.H.S.} = 2^2 = 4$$

$$\text{R.H.S.} = (-2)^2 = 4$$

$\therefore f(x_1) = f(x_2)$ but $x_1 \neq x_2$

\therefore not one-to-one

b) f is on-to $\longleftrightarrow \forall y \in \mathbb{Z}^+ \exists x \in \mathbb{Z} \quad y = f(x)$

$$y = x^2$$

$$x = \pm \sqrt{y}$$

for $y = 2 \in \mathbb{Z}^+$

$$x \notin \mathbb{Z}$$

\therefore not on-to

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Q1. (6marks) Prove that for all positive integers a, b, c if a divides b and b divides c then a divides c.

$$\forall a, b, c \in \mathbb{Z}^+ \quad b = ak_1 \wedge c = bk_2 \rightarrow c = ak$$

To assume: $b = ak_1$, $c = bk_2$, $a, b, c \in \mathbb{Z}^+$, $k_1, k_2 \in \mathbb{Z}$

To conclude: $c = ak$

$$c = bk_2 \Rightarrow c = (ak_1)k_2 \quad \text{(from assumption)}$$

$$c = a(k_1k_2)$$

$$c = ak, \quad k \in \mathbb{Z}, \quad k = k_1k_2$$

Hence proved.

Q2. (4marks) Is the following true? If yes, then prove it. If false, give a counter example.

For all integers n, $\frac{n^2}{n^2+1}$ is not an integer.

false.

counter example:

$$\text{for } n = 0 \in \mathbb{Z} \Rightarrow \frac{n^2}{n^2+1} = \frac{0}{0+1} = 0 \in \mathbb{Z}$$